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Cosmic Numbers: A Physical Classification for Cosmological Models

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We introduce the notion of the cosmic numbers of a cosmological model, and discuss how they can be used to naturally classify models according to their ability to solve some of the problems of the standard cosmological model.

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The idea of possible time and space variations of the fundamental 'constants' of nature has attracted a lot of interest in recent times. This has been powered, on the observational side, by the observational hints for possible variations of the fine structure constant, α [1], and the ratio of the proton and electron masses, $\mu = m_p/m_e$ [2], detected by spectroscopy of low-redshift astrophysical sources. A further, arguably less robust hint also comes from the Oklo natural reactor [3], while the Cosmic Microwave Background and Big Bang Nucleosynthesis provide relatively weaker bounds, though at much higher redshifts [4, 5, 6]. Further constraints are also discussed in [7].

On the theoretical side there have been several claims that some of the major problems of the standard cosmological model [8] could be solved in a varying α theory if α was smaller in the past, or alternatively in so-called varying speed of light (VSL) theories [9, 10, 11, 12] or in bi-metric theories [13, 14, 15]. At a different level, varying fundamental constants are ubiquitous in theories with additional spacetime dimensions [16].

There has also been some controversy on whether or not it is always possible to identify which of the fundamental 'constants' are varying [17]. It is worth pointing out that some of this this discussion is rather 'academic' in scope, often relying on rather contrived interpretations of particular definitions or concepts. Here we aim to clarify this debate, in the spirit of [18] by providing a physical classification of cosmological models according to the behaviour of their characteristic cosmic numbers (which will be defined below). From a practical (experimental) point of view, it should be obvious that experiments can only measure dimensionless combinations of fundamental parameters, despite some attempts to classify theories as varying electric charge (e) or varying speed of light (c) depending on simplicity arguments [19, 20]. This means that when measuring a dimensional parameter *one should always specify the choice of units in which it is measured.*

Let us now focus on cosmology, and assume that the cosmological principle holds. Then there is always a class of (co-moving) observers for which the universe is homogeneous and isotropic. This space-time is therefore described by a Friedmann-Robertson-Walker metric,

$$ds^2 = -c^2dt^2 + a^2(t)\left(\frac{dr^2}{1-kr^2} + r^2 d\Theta_2^2\right),$$

regardless of any possible modifications to the dynamics of our theory of gravity. We expect that such modifications will appear naturally in the context of the possible variations of the fundamental 'constants' of nature.

This means that, quite generally, cosmology provides us with two 'natural' units: one unit of length, *the curvature scale*, $\ell_c \equiv a|k|^{-1/2}$, and one unit of time, *the Hubble time*, $H^{-1} \equiv a/(da/dt)$. Hence, the natural way (in a cosmological sense) of measuring the speed of light in a given cosmological model is to express it in these units. We shall call this the *expansion number*, and define it as

$$C_e \equiv \frac{c|k|^{1/2}}{aH}.$$

Note that this is a dimensionless quantity. With this definition, we immediately see that 'varying cosmic speed' (VCS) theories are generic [18]. For example, in the standard cosmological model one has $C_e \propto t^{1/2}$ during the radiation dominated era, $C_e \propto t^{1/3}$ during the matter dominated era, $C_e = const.$ during a curvature dominated era, and $C_e \propto \exp(-Ht)$ if the cosmological constant dominates.

One can easily show with all generality (that is, assuming only the cosmological principle) that the resolution of the horizon and flatness problems depends naturally on the behaviour of the expansion number C_e, rather than on the behaviour of c in units of $e^2/\hbar$ (in which case it would be related to the variation of α). Before discussing the general case, we notice as an aside that the flatness problem is in fact trivially solved if the *standard* Friedmann equation holds, since in that case the expansion number defined above reduces to

$$C_{e,st} = |\Omega - 1|^{1/2}.$$

However, the main point we wish to clarify in this letter is that regardless of the explicit expression for C_e

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(which will of course be model-dependent), it will still be true that the behaviour of this parameter will determine whether or not the horizon and flatness problems can be solved.

Indeed, a solution to the flatness problem depends only on the behaviour of C_e, independently of any modifications to the Friedmann equation. This is easy to understand given that the flatness problem can be solved if our theory explains why the current Hubble radius, $c_0 H_0^{-1}$, is much smaller than (or of the same order of) the curvature scale, $a_0|k|^{-1/2}$. On the other hand, a solution to the horizon problem can be achieved by having a period in the history of the Universe in which the scale factor, a, grows faster than the Hubble radius, cH^{-1}. In both cases, we assume that the Cosmological Principle holds for a region with size at least equal to the initial Hubble radius. Note that the same is required in inflationary models: assuming that General Relativity holds, the vacuum energy density which drives inflation must not be very inhomogeneous over a region of several Hubble radii if inflation is to begin.

In summary, what this means is that both the horizon and flatness problems can be solved if

$$\frac{d}{dt}C_e < 0\,,$$

for a suitable (that is, large enough) period of time in the early Universe. We call these decreasing cosmic speed (DCS) theories, as opposed to increasing cosmic speed (ICS) ones. Therefore we see that both the standard inflationary paradigm, the deflationary phase in the pre-Big-Bang scenario [21] and some of the so-called VSL theories are examples of DCS theories, and they are therefore capable of solving these cosmological problems. On the other hand, the standard cosmological model is an example of an ICS theory, and hence can not solve them.

We emphasize that this classification is entirely independent of what is happening to the dimensionful quantities in the model—an obvious point, since one can always change this behaviour by a suitable re-definition of our units of measurement (for length, time, and energy). But more to the point, this is *also generically independent* of the behaviour of dimensionless particle physics quantities in the model.

It is also worth stressing that the existence of a period where the above condition holds is *necessary* but may not be sufficient for these problems to be solved. While satisfying the above equation, the universe will be getting closer to flatness, but it could for example re-collapse before such a period begins. This is of course a well known problem in other contexts, such as inflation in closed universes.

Other cosmic numbers can be similarly defined. For example, the *rotation number*,

$$C_r \equiv w/H\,,$$

and the *shear number*,

$$C_s \equiv \sigma/H\,,$$

are dimensionless measures of the angular velocity of the universe, w, and the cosmic shear, σ, respectively. The corresponding rotation and shear problems are related to the un-natural smallness of these numbers [22]. Inflation is of course an example of a theory where these problems can be solved.

Hence the message so far is that the cosmological problems can not in general be solved by an arbitrary modification of dimensionless particle physics parameters. In general a solution to these problems will require an appropriate modification to dimensionless variables involving various cosmological quantities.

On the other hand, there are 'hybrid' problems whose solution will involve combinations of cosmological quantities and the so-called 'fundamental constants of nature'. For example, explaining why the Hubble time is much larger than the Planck time requires a period where

$$\frac{d}{dt}\left(\frac{G\hbar H^2}{c^5}\right) < 0\,.$$

Notice that this involves not only Planck's constant but also Newton's constant (G), and hence the scale of gravity, which can in some cases have a non-trivial dynamics (brane world models being a topical example). Furthermore, this also means that the flatness and oldness problems are in general *different* problems, and may therefore be solved in some theories by quite different mechanisms. Only if the *standard* Friedmann equation is valid do they become the same.

Finally, we note that there are other problems of the standard cosmological model (such as the seed fluctuations and cosmological constant problems) that can not be satisfactorily solved in this way. Rather, their solution is strongly dependent on the detailed dynamics of the specific model in question, and therefore no generic classification can be given.

[1] J. K. Webb et al., Phys. Rev. Lett. **87**, 091301 (2001), astro-ph/0012539.

[2] A. V. Ivanchik, E. Rodriguez, P. Petitjean, and D. A. Varshalovich, Astron. Lett. **28**, 423 (2002), astro-ph/0112323.

[3] Y. Fujii et al. (2002), hep-ph/0205206.

[4] P. P. Avelino, C. J. A. P. Martins, and G. Rocha (2000), astro-ph/0001292.

[5] P. P. Avelino et al., Phys. Rev. **D64**, 103505 (2001), astro-ph/0102144.

[6] C. J. A. P. Martins et al., Phys. Rev. **D66**, 023505 (2002), astro-ph/0203149.

[7] J.-P. Uzan (2002), hep-ph/0205340.

[8] E. W. Kolb and M. S. Turner (1990), Redwood City, U.S.A.: Addison-Wesley.

[9] J. W. Moffat, Int. J. Mod. Phys. **D2**, 351 (1993), gr-qc/9211020.

[10] A. Albrecht and J. Magueijo, Phys. Rev. **D59**, 043516 (1999), astro-ph/9811018.

[11] J. D. Barrow and J. Magueijo, Phys. Lett. **B443**, 104 (1998), astro-ph/9811072.

[12] J. D. Barrow and J. Magueijo, Class. Quant. Grav. **16**, 1435 (1999), astro-ph/9901049.

[13] B. A. Bassett, S. Liberati, C. Molina-Paris, and M. Visser, Phys. Rev. **D62**, 103518 (2000), astro-ph/0001441.

[14] P. P. Avelino and C. J. A. P. Martins, Phys. Rev. Lett. **85**, 1370 (2000), astro-ph/0002413.

[15] P. P. Avelino and C. J. A. P. Martins, Phys. Lett. **B516**, 191 (2001), astro-ph/0006303.

[16] C. J. A. P. Martins (2002), astro-ph/0205504.

[17] M. J. Duff, L. B. Okun, and G. Veneziano, JHEP **03**, 023 (2002), physics/0110060.

[18] P. P. Avelino and C. J. A. P. Martins, Phys. Lett. **B459**, 468 (1999), astro-ph/9906117.

[19] J. Magueijo, J. D. Barrow, and H. B. Sandvik (2002), astro-ph/0202374.

[20] P. C. W. Davies, T. M. Davis, and C. H. Lineweaver, Nature **418**, 602 (2002).

[21] M. Gasperini and G. Veneziano, Mod. Phys. Lett. **A8**, 3701 (1993), hep-th/9309023.

[22] E. F. Bunn, P. Ferreira, and J. Silk, Phys. Rev. Lett. **77**, 2883 (1996), astro-ph/9605123.